THE COMPLEXITY OF FINITE-VALUED CSPS

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Let Γ be a set of rational-valued functions on a fixed finite domain; such a set is called a *finite-valued constraint language*. The valued constraint satisfaction problem, VCSP(Γ), is the problem of minimising a function given as a sum of functions from Γ.

Using ideas from the algebraic programme for CSPs, a dichotomy theorem has recently been established [2] with respect to exact solvability for *all* finite-valued languages defined on domains of *arbitrary* finite size. The classification is based on a generalisation of submodularity, called *fractional polymorphisms*, and on previous results characterising to which extent linear programming relaxations can be used to solve instances exactly [1, 3].

Theorem 0.1. *Let Γ be a core finite-valued language of function defined on a finite set D. Then, either*

- *VCSP(Γ) is solvable by linear programming; or*
- *VCSP(Γ) is NP-hard.*

Theorem 0.1 shows that there is a single algorithm for all polynomial-time solvable cases and a single reason for intractability. The boundary between P and NP, depending on Γ, can be given explicitly. The result also demonstrates that, for exact solvability of VCSPs, the basic linear programming relaxation suffices and semidefinite programming relaxations do not add any additional power.

Theorem 0.1 generalises *all previous partial classifications* of finite-valued languages: the classifications of $\{0, 1\}$-valued languages (i.e., Max CSP) on two-element, three-element, and four-element domains obtained by Creignou [JCSS'95], Jonsson et al. [SICOMP'06], and Jonsson et al. [CP'11], respectively; the classification of $\{0, 1\}$-valued languages containing all unary functions obtained by Deineko et al. [JACM'06]; the classifications of finite-valued languages on two-element and three-element domains obtained by Cohen et al. [AIJ'06] and Huber et al. [SODA'13], respectively; the classification of finite-valued languages containing all $\{0, 1\}$-valued unary functions obtained by Kolmogorov and Živný [SODA'12]; and the classification of Min-0-Ext problems obtained recently by Hirai [SODA'13].

This is joint work with Stanislav Živný (Warwick, UK).

References

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